

HEAT EXCHANGERS

L. L. Vasil'ev and S. V. Konev

UDC 536.24

Many articles have been published in recent years on the subject of heater tubes or, more properly, tubular heat exchangers (Fig. 1). These devices are of interest to engineers on account of their design simplicity and their effectiveness in use as very high conductivity heat transmitters. They operate on the principle that the thermal flux is transmitted through evaporation of a liquid in the evaporator section and subsequent condensation of the vapor in the condenser section of the tube. A steady state in this process is attained by having the vapor flow from evaporator to condenser through the center duct, while the liquid condensate returns from condenser to evaporator through a capillary wick lining the tube wall. In the general classification, tubular heat exchangers are devices with the ratio $L/d > 10$, i. e., with a length much greater than the diameter. In many cases, however, $L/d < 10$ and such devices are more properly called vaporizing chambers.

According to the temperature range over which modern tubular heat exchangers and vaporizing chambers operate, one distinguishes three class of such devices:

1. Low-temperature tubular heat exchangers and vaporizing chambers for $t < 400^\circ\text{K}$: the heat is carried here by cryogenic media (Freon, nitrogen, hydrogen, neon, ammonia, etc.), water, solutions of salts, etc.
2. Medium-temperature tubular heat exchangers ($400 < t < 1200^\circ\text{K}$): liquid metals such as sodium, rubidium, cesium, etc. are used here.
3. High-temperature tubular heat exchangers ($1200 < t < 2000^\circ\text{K}$): lithium, gallium, lead, indium, silver, etc. are used here.

Vaporizing chambers are commonly used in the low-temperature range, while tubular heat exchangers are used primarily in the medium- and high-temperature ranges.

The maximum thermal power transmittable through a tubular heat exchanger is that at which desiccation of the porous wick in the evaporator section begins while the wall temperature increases sharply. The evaporator desiccation may be due to an inadequate transmittability of the porous wick or due to a plugging up of the pores by various products of interaction between the heat carrier and the wick or the tube material.

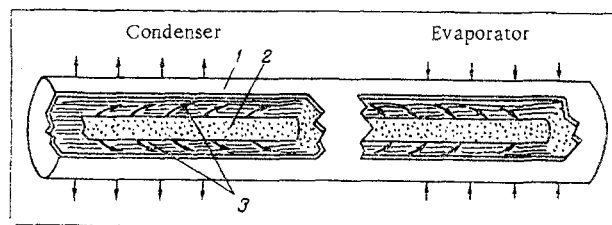


Fig. 1. General view of a tubular heat exchanger: 1) outer shell; 2) vapor stream; 3) wick saturated with liquid.

Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 20, No. 3, pp. 550-566, March, 1971. Original article submitted May 6, 1970

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In order to fully describe the operating principle of a tubular heat exchanger, it is necessary to first analyze the processes taking place in the evaporator and in the condenser as well as in the thermal insulation and in the wick.

The mass transfer in a tubular heat exchanger is effected by a pressure gradient resulting from a temperature gradient. The transfer of thermal energy occurs as a result of phase transitions during evaporation and condensation.

For a steady-state operation of a tubular heat exchanger, the pressure drop around a closed path inside the tube must be $\Sigma P = 0$, i. e.,

$$(P_{V(e)} - P_{V(c)}) + (P_{V(c)} - P_{L(c)}) + (P_{L(c)} - P_{L(e)}) + (P_{L(e)} - P_{V(c)}) = 0.$$

At the same time, it is necessary that

$$\Delta P_{L(f)} + \Delta P_{L(B)} + \Delta P_{V(a.r+in)} \leq \Delta P_{cap}.$$

The action of capillary forces is described by the Laplace-Young equation:

$$\Delta P_{cap} = \sigma \left(\frac{1}{R'} - \frac{1}{R''} \right),$$

where R' and R'' are the curvature radii of the three-dimensional meniscus surface.

If the liquid wets the wick, then the contact angle is less than 90° and, denoting by R_c and R_e the curvature radii of the liquid-vapor interface in the condenser and in the evaporator respectively,

$$P_{V(c)} - P_{L(c)} = \frac{2\sigma}{R_c},$$

$$P_{L(e)} - P_{V(e)} = -\frac{2\sigma}{R_e},$$

$$\Delta P_{cap} = \frac{2\sigma}{R_c} - \frac{2\sigma}{R_e}.$$

When a tubular heat exchanger operates under optimal conditions, the radius of the vapor-liquid interface in the condenser tends toward infinity $R_c \rightarrow \infty$, i. e., a thin liquid film forms on a flat surface. Then

$$\Delta P_{cap} = -\frac{2\sigma}{R_e} = -\frac{2\sigma}{R_{min}}.$$

The pressure drop ΔP_V across the vapor phase in a tubular heat exchanger is produced by friction and inertia forces acting when vapor flows.

The flow of vapor in the evaporator and in the condenser of a tubular heat exchanger can be characterized by the radial velocity U_r and the axial velocity U_x . In the thermally insulated section of a tube we will consider only the axial velocity U_x .

We will treat a tubular heat exchanger as a cylinder.

The steady-state Navier-Stokes equations in cylindrical coordinates are

$$U_r \frac{\partial U_x}{\partial r} + U_x \frac{\partial U_x}{\partial x} = -\frac{1}{\rho} \cdot \frac{\partial P}{\partial x} + \nu \left[\frac{1}{r} \cdot \frac{\partial}{\partial r} \left(r \frac{\partial U_x}{\partial r} \right) + \frac{\partial^2 U_x}{\partial x^2} \right], \quad (1)$$

$$U_r \frac{\partial U_r}{\partial r} + U_x \frac{\partial U_r}{\partial x} = -\frac{1}{\rho} \cdot \frac{\partial P}{\partial r} + \nu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \cdot \frac{\partial}{\partial r} (rU_r) \right) + \frac{\partial^2 U_r}{\partial x^2} \right]. \quad (2)$$

The continuity equation is

$$\frac{\partial(rU_x)}{\partial x} + \frac{\partial(rU_r)}{\partial r} = 0. \quad (3)$$

Bearing in mind that the length of a tubular heat exchanger is much greater than its radius, that the tube geometry is fixed, and that the thermophysical properties of the heat carrier are constant, we can simplify this system of equations considerably:

$$\frac{\partial P}{\partial x} = -\rho \left(U_r \frac{\partial U_x}{\partial r} + U_x \frac{\partial U_x}{\partial r} \right) + \mu \left(\frac{1}{r} \cdot \frac{\partial U_x}{\partial r} + \frac{\partial^2 U_x}{\partial r^2} \right), \quad (4)$$

$$\frac{\partial P}{\partial r} = -\rho \left(U_r \frac{\partial U_r}{\partial r} + U_x \frac{\partial U_r}{\partial x} \right) + \mu \left(\frac{\partial^2 U_r}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial U_r}{\partial r} - \frac{U_r}{r^2} \right), \quad (5)$$

$$\frac{\partial(rU_x)}{\partial x} + \frac{\partial(rU_r)}{\partial r} = 0. \quad (6)$$

The boundary conditions for solving this system of equations can be specified as follows:

at the tube wall

$$\begin{aligned} U_{r(e)} &= -U_R, \\ r = R, \quad U_x &= 0, \quad U_{r(i)} = 0, \\ U_{r(c)} &= U_R, \end{aligned} \quad (7)$$

along the tube axis

$$r = 0, \quad U_r = 0, \quad U_x = 0 \quad \text{at} \quad x = 0. \quad (8)$$

If we denote the length of the evaporator by l_e , the length of the thermally insulated section by l_i , and the length of the condenser by l_c , then

$$l_1 = l_e + l_i; \quad l_2 = l_1 + l_c. \quad (9)$$

The system of partial differential equations shown here was transformed in [1] into ordinary differential equations by introducing a flow function ψ which satisfies the continuity condition

$$rU_x = \partial\psi/\partial r \quad \text{and} \quad -rU_r = \partial\psi/\partial x, \quad (10)$$

and assuming that

$$\psi = \left[C_1 + C_2 \frac{x}{R} \right] f(\xi), \quad 0 \leq x \leq l_e, \quad (11)$$

where $\xi = (r/R)^2$.

Constants C_1 and C_2 are determined from the boundary conditions. As a result, we have for the flow function

$$\psi = \frac{RxU_R}{f(\xi)_{\frac{r}{R}=1}} f(\xi). \quad (12)$$

Then

$$U_x = \frac{2}{f(\xi)_{\frac{r}{R}=1}} U_r \frac{x}{R} f'(\xi), \quad (13)$$

$$U_r = \frac{1}{\sqrt{\xi}} \cdot \frac{U_r}{f(\xi)_{\frac{r}{R}=1}} f'(\xi). \quad (14)$$

Inserting these expressions for the velocities (13) and (14) into the Navier–Stokes equation, we obtain

$$\frac{\partial P}{\partial x} = -\mu \frac{4Re_r}{f(\xi)_{\frac{r}{R}=1}} \cdot \frac{x}{R^4} \left[\frac{U_r R}{f(\xi)_{\frac{r}{R}=1}} (f'' - ff'') - 2\mu (\eta f''' f'') \right], \quad (15)$$

$$\frac{\partial P}{\partial \xi} = \frac{U_r \rho}{f(\xi) \frac{r}{R}=1} R \left[\frac{U_r}{f(\xi) \frac{r}{R}=1} R \left(\frac{f^2}{\xi^2} - \frac{2ff'}{\xi} - 2vf'' \right) \right]. \quad (16)$$

Here $Re_r = U_r R / \nu$ is the Reynolds number for radial flow.

Differentiating Eq. (15) with respect to ξ will yield

$$\frac{d}{d\xi} \left[\frac{U_r R}{f(\xi) \frac{r}{R}=1} (f'^2 - ff'') - 2v(\eta f'' - f'') \right] = 0. \quad (17)$$

Integrating this equation yields

$$\frac{U_r R}{f(\xi) \frac{r}{R}=1} (f'^2 + ff'') - 2v(\eta f''' + f''') = C' \quad (18)$$

or, using the Reynolds number Re_r ,

$$f'^2 - ff'' - 2f(\xi) \frac{r}{R}=1 \cdot \frac{1}{Re_r} (\eta f''' + f''') = C''. \quad (19)$$

The solution of this third-order nonlinear differential equation is given in [2, 3, 4] for the boundary conditions

$$f(0) = 0, \quad f'(1) = 0, \quad f''(0) = 0, \quad f(1) = 0.5. \quad (20)$$

In [1] is given the solution for $Re_r \gg 1$:

$$f(\xi) = \frac{1}{2} \sin \frac{\pi}{2} \eta + \frac{1}{Re_r} \left[0.727 \cos \frac{\pi}{2} \eta + 1.525 \left(\frac{2}{\pi} \sin \frac{\pi}{2} \eta - \eta \cos \frac{\pi}{2} \eta \right) + \dots - \left(1.5 - \frac{4}{\pi^2} 1.33 \sin \frac{\pi}{2} \eta \right) \right]. \quad (21)$$

For liquids whose vapors have a low dynamic viscosity Eq. (13) can, to the first approximation, be reduced to an equation of the type

$$f'^2 - ff'' = C''. \quad (22)$$

The solution to this equation with the boundary conditions (7) and (8) is

$$f = \frac{1}{2} \sin \frac{\pi}{2} \xi. \quad (23)$$

Combining Eq. (15) and (16) yields the following expression for the pressure drop across the vapor phase in a tube:

$$\Delta P_V = P(0, r) - P(x, r) = 8\rho_V U_r^2 \left(\frac{x}{R} \right)^2 \left(\frac{1.325}{Re_r} + 0.617 \right). \quad (24)$$

In the evaporator

$$\Delta P_{V(e)} = 8\rho_V U_r^2 \left(\frac{l_e}{R} \right)^2 \left(\frac{1.325}{Re_r} + 0.617 \right). \quad (25)$$

The ratio of radial velocity U_r to mean axial velocity U_x of the vapor at the evaporator outlet can be obtained from the law of energy conservation:

$$\frac{U_r}{U_x} = \frac{1}{2} \cdot \frac{R}{l_e},$$

and analogously

$$\frac{Re_r}{Re_x} = \frac{1}{2} \cdot \frac{R}{l_e}.$$

Letting

$$Re = \frac{U_x R}{\nu},$$

we have in the evaporator

$$\Delta P_{V(e)} = \left(1.234 + \frac{5.3}{Re_x} \cdot \frac{l_e}{R} \right) \rho_V U_x^2 \quad (26)$$

for $Re \gg 1$.

In [5] the pressure drop across the evaporation zone has been determined as

$$\Delta P_{V(e)} = \frac{8\eta U_x l_e}{R^2} \left[1 + \frac{7}{9} Re_r - 3.16 + 0.056C^2 \right] \left(1 - \frac{x^2}{l_e^2} \right). \quad (27)$$

In the thermally insulated zone the hydrodynamic flow of vapor is analogous to the gas flow in a tube with rough walls. This process can be described, in the case of laminar flow, by the Poiseuille law:

$$\Delta P_{V(i)} = \frac{1}{2} \rho_V U_x^2 \left(16 \frac{x}{R Re_x} \right). \quad (28)$$

According to [5], the pressure drop across the vapor phase in the thermally insulated zone is

$$\begin{aligned} \Delta P_{V(i)} = & \frac{16U_x \eta l_i}{Re_x^2} \left[1 + \frac{0.1 Re_r}{5 Re_r + 18} \cdot \frac{1 - \exp\left(-\frac{30l_i}{R Re_x}\right)}{l_i (R Re_x)^{-1}} \right] \\ & - \frac{16U_x \eta (x - l_e)}{Re_x^2} \left[1 + \frac{1.106 Re_r}{18 + 5 Re_r} \cdot \frac{1 - \exp\left(-\frac{30(x - l_e)}{R Re_x}\right)}{(x - l_e) (R Re_x)^{-1}} \right]. \end{aligned} \quad (29)$$

In the condensation zone of a tube the condensation process is to some extent similar to suction through a porous wall. During condensation of vapor from a laminar boundary layer, the pressure drop is much smaller than in the same tube without condensation or suction and, therefore, one may disregard it and consider the pressure in the condenser to be constant and equal to the vapor pressure at the condenser inlet.

In this way, the pressure drop across the vapor phase in a tube with a laminar flow can be expressed as follows:

$$\Delta P_V = \Delta P_{V(e)} + \Delta P_{V(i)} + \Delta P_{V(c)} = \left(1.234 + \frac{5.3}{Re_x} \cdot \frac{l_e}{R} \right) \rho_V U_x^2 + 0.5 \rho_V U_x^2 \left(16 \frac{l_i}{R Re_x} \right) \quad (30)$$

or

$$\begin{aligned} \Delta P_V = & \frac{8U_x \eta l_e}{R^2} \left[1 + \frac{7}{9} Re_r - 3.16 + 0.056C^2 \right] + \frac{16U_x \eta l_i}{R^2} \left[1 + \frac{0.106 Re_r}{18 + 5 Re_r} \frac{1 - \exp\left(-\frac{30l_i}{R Re_x}\right)}{l_i (R Re_x)^{-1}} \right] \\ & - \frac{16U_x \eta (x - l_e)}{R^2} \left[1 + \frac{1.106 Re_r}{18 + 5 Re_r} \frac{1 - \exp\left(-\frac{30(x - l_e)}{R Re_x}\right)}{(x - l_e) (R Re_x)^{-1}} \right]. \end{aligned} \quad (31)$$

To a rough approximation for long tubes, when $l_i \gg l_e$, one may disregard the pressure drop in the evaporator as well as in the condenser, and then

$$\Delta P_V = \Delta P_{V(i)}$$

If the Poiseuille law is assumed valid here, then

$$\Delta P_V = - \frac{8\mu l}{\pi R^4} j. \quad (32)$$

For turbulent vapor flow in a tubular heat exchanger the pressure drop is determined as follows: in the evaporator

$$\Delta P_{V(e)} = 4.45 U_x \rho_V \frac{U_x l_e}{R}; \quad (33)$$

and, considering that

$$\frac{U_r}{U_x} = 0.5 \frac{R}{l_e},$$

then

$$\Delta P_{V(i)} = 2.23 \rho_V \bar{U}_x^2; \quad (34)$$

in the thermally insulated tube section, according to the Blasius formula,

$$\Delta P_{V(i)} = 0.0107 \frac{\mu_V^{1/4} l_i}{\rho_V R^{19/4}}; \quad (35)$$

and in the condenser

$$\Delta P_{V(c)} \approx 0.$$

Therefore, the total pressure drop across a tube with a turbulent vapor flow is

$$\Delta P_V = 4.45 \rho_V \bar{U}_x^2 \frac{U_r}{\bar{U}_x} \cdot \frac{l_e}{R} + 0.0107 \frac{\mu_V^{1/4} l}{\rho_V R^{19/4}}. \quad (36)$$

Until now we have considered only the effect of friction forces on the hydrodynamics of vapor flow in heater tubes. When the velocity of vapor approaches the velocity of sound, however, it becomes necessary to also consider the forces of inertia.

The velocity of sound presents probably also one of the basic limitations on the transmission of large thermal fluxes along a tube, since shock waves may be generated as the velocity of sound is approached and the passage at the evaporator outlet may become plugged.

The pressure drop across the wick is found according to the formula:

$$\Delta P_L = k l j \frac{\mu_L}{\rho_L}. \quad (37)$$

At the present time, porous wicks are designed in the form of metallic mesh pieces braided one to another or in the form of metal-ceramic sleeveings (such wicks are used especially in low- and medium-temperature tubular heat exchangers). One must, therefore, know the conditions of liquid flow through a porous wick.

In several articles on the subject of liquid flow through a porous material this flow has been assumed to be laminar (Poiseuillian) through the pores and the formula for laminar flow of a viscous liquid in a cylindrical tube

$$\Delta P_L = \frac{8 \nu_L}{\pi R^4} l j \quad (38)$$

has been used.

This is not quite correct. During capillary suction the velocity at the wall can be considerably different from zero.

The flow of liquid through a porous body due to a gradient of total pressure is more correctly described by the generalized Darcy law. For the one-dimensional case this law can be written as

$$j_L = -k(\theta) \text{grad } \Phi, \quad (39)$$

and for a horizontal flow of a liquid or for the flow of a weightless liquid this becomes simply

$$j_L = -\frac{k \rho_L}{\mu_L} \cdot \frac{dP}{dx}. \quad (40)$$

In actual heater tubes the assumption of a one-dimensional flow through the wick is by far not always admissible. Particularly for thin porous wicks, when the vapor velocity is high, it becomes necessary to account for its interaction with the liquid near the surface (wave formation at the surface) and this affects the velocity distribution in the liquid across the wick section.

For this reason, one must distinguish at least two velocity components in the liquid:

$$U_x = -k \frac{\partial P}{\partial x}; \quad U_y = -k \frac{\partial P}{\partial y}.$$

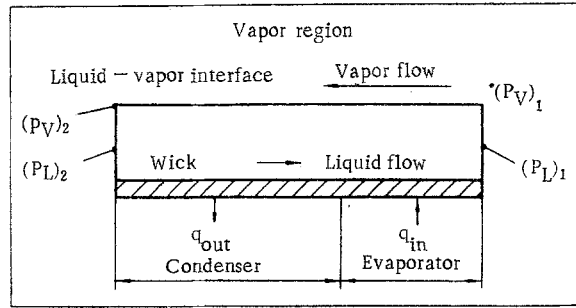


Fig. 2. Simplified model of a heater tube.

When the wick is thin, $x \ll R$, then the flow of liquid through it is described by the Poisson equation:

$$\frac{\partial U_x}{\partial x} + \frac{\partial U_r}{\partial y} = k \left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} \right). \quad (41)$$

This effect is disregarded, however, in wick design calculations. The most commonly used equation of mass transfer through a porous body is [6]:

$$\frac{\partial \theta}{\partial \tau} + \tau' \frac{\partial^2 \theta}{\partial \tau^2} = \text{div} (a_m \text{grad } \theta), \quad (42)$$

where $a_m = k(\partial \Phi / \partial \theta)$ is the hydraulic diffusivity,

$$\Phi = h + \int_{P_s}^P \frac{dP}{\rho_L}.$$

The use of Eq. (42) is especially important for analyzing thermal shock processes in tubular heat exchangers placed in the field of gravity, when the supplied heat causes an intensive evaporation of the liquid and when the condensate flows through an unsaturated porous wick at a finite velocity.

In several cases where the term $\tau' (\partial^2 \theta / \partial \tau^2)$ is negligible Eq. (42) can be simplified into

$$\frac{\partial \theta}{\partial \tau} = \text{div} [a_m \text{grad } \theta] + \frac{\partial k(\theta)}{\partial x}. \quad (43)$$

In order to solve this equation, it is necessary to know the quantities $a_m = f(\theta)$ and $k = f(\theta)$ for any specific porous material.

The nonlinear differential equations (42) and (43) are very difficult to solve not only by analytical but also by numerical methods, because $a_m(\theta)$ and $k(\theta)$ are highly nonlinear and also because there is a large difference between the initial and the subsequent rates of suction. A numerical solution of the problem concerning two-dimensional mass transfer through porous media has been shown in [7].

Often an exponential approximation of $a_m(\theta)$ is used. Equation (43) can then be rewritten as

$$\frac{d\theta}{d\tau} = \frac{\partial}{\partial x} \left[c \exp \theta \frac{\partial \theta}{\partial x} \right] \quad (44)$$

and, therefore, be represented in the form:

$$\frac{\Delta x^2}{\Delta \tau} (\theta_j^{\tau+1} + \theta_j^\tau) = \exp(\theta_{j+1}^\tau) (\theta_{j+1}^\tau - \theta_j^\tau) - \exp(\theta_{j-1}^\tau) (\theta_j^\tau - \theta_{j-1}^\tau), \quad (45)$$

making it possible to calculate $\theta_j^{\tau+1}$.

In order to describe the process of mass transfer through a porous wick of a heater tube, it suffices to apply the equations of filtration to the transfer of a liquid

$$\text{div} \left[\frac{\rho k}{\eta} (\nabla P + \rho \nabla h) \right] = \Pi \frac{\partial \rho}{\partial \tau}. \quad (46)$$

where $a_m(\theta)$ and $k(\theta)$ are assumed constant.

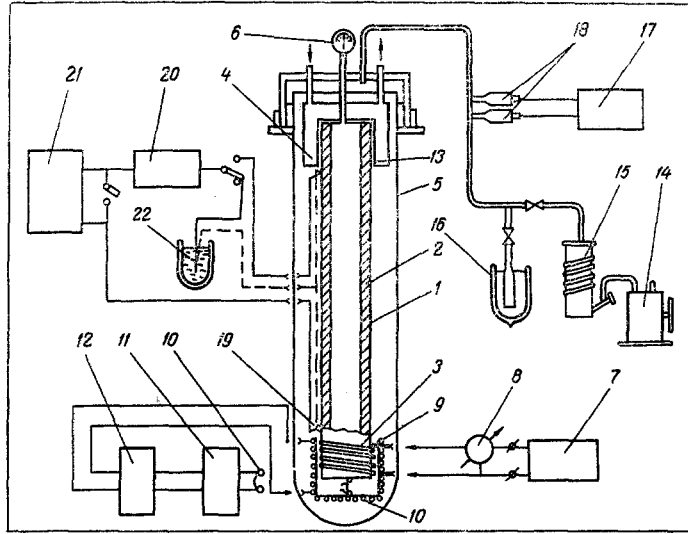


Fig. 3. Schematic diagram of the experimental apparatus: 1) glass housing; 2) wick; 3) heater; 4) cooler; 5) glass cover; 6) manovacuumeter; 7) electric power supply; 8) wattmeter; 9) adiabatic case; 10) differential thermocouple; 11) photo-compensation amplifier; 12) power amplifier; 13) thermal insulation; 14) prevacuum pump; 15) vacuum pump; 16) nitrogen trap; 17) vacuumeter; 18) vacuumeter pickoffs; 19) thermocouples; 20) low-resistance potentiometer; 21) automatic recorder; 22) thermocouple reference junction.

For a homogeneous liquid in an isotropic porous material this equation in cylindrical coordinates becomes

$$\frac{1}{r} \cdot \frac{\partial}{\partial r} \left[\frac{r\rho k}{\eta} \left(\frac{\partial P}{\partial r} + \rho \frac{\partial h}{\partial r} \right) \right] + \frac{1}{r^2} \cdot \frac{\partial}{\partial \psi} \left[\frac{\rho k}{\eta} \left(\frac{\partial P}{\partial \psi} + \rho \frac{\partial h}{\partial \psi} \right) \right] + \frac{\partial}{\partial x} \left[\frac{\rho k}{\eta} \left(\frac{\partial P}{\partial x} + \rho \frac{\partial h}{\partial x} \right) \right] = \Pi \frac{\partial \rho}{\partial \tau}, \quad (47)$$

where ψ is the suction potential.

If density ρ depends on pressure only, then potential Φ and Eq. (46) can be used:

$$\operatorname{div} \left(\frac{\rho^2 k}{\eta} \nabla \Phi \right) = \Pi \frac{\partial \rho}{\partial \tau}. \quad (48)$$

For steady-state conditions, with k and η constant, Eq. (48) becomes the Laplace equation:

$$\nabla^2 \Phi = 0. \quad (49)$$

Its solution is

$$\Phi - \Phi_0 = Cx. \quad (50)$$

In order to optimize the performance of a tubular heat exchanger, one often designs the wicks with structural-mechanical properties which vary along a space coordinate, since optimal condensation in the condenser and optimal evaporation in the evaporator require different pore sizes. For variable permeability which is a function of a coordinate, then, the Laplace equation becomes

$$\frac{d}{dx} \left(k \frac{d\Phi}{dx} \right) = 0. \quad (51)$$

If we introduce a new space variable β , then Eq. (51) can be reduced to (49) and

$$\Phi - \Phi_0 = c\beta; \quad \beta = \int_{x_0}^x \frac{dx}{k}. \quad (52)$$

TABLE 1. Calculated Values of Coefficient K for Low-Boiling Liquids

Heat carrier	T _{boil} , °K	ρ, g/cm ³	η, g/cm·sec	σ, dyne/cm	r', erg/g	k = p σ r' ¹ / η, g/sec ³
Ammonia -30°C	239,76	0,702	83·10 ⁻⁶	32,8	1358 ·10 ⁷	3635·10 ¹³
Water 20°C	373,16	0,998	1005·10 ⁻⁶	73	2450 ·10 ⁷	1780·10 ¹²
Methanol 20°C	337,67	0,795	5840·10 ⁻⁶	22,6	1177 ·10 ⁷	36·10 ¹²
Acetone 20°C	329,26	0,790	3250·10 ⁻⁶	23,7	522 ·10 ⁷	30,4·10 ¹²
Ethanol 20°C	351,46	0,789	6950·10 ⁻⁶	22,8	919,6·10 ⁷	24,4·10 ¹²
H ₂ 20°C	20,28	0,0714	140·10 ⁻⁶	2,52	454	521·10 ⁸
He I 3°K	4,22	0,1425	33·10 ⁻⁶	0,22	23,6	22,4·10 ⁸
He II 2°K	—	0,1468	9,4·10 ⁻⁶	0,31	23,3	113·10 ⁸
He II 1,5°K	—	0,1468	1,74·10 ⁻⁶	0,33	22,5	606·10 ⁸
Freon-12 0°C	243,2	1,394	2750·10 ⁻⁶	12	154,5·10 ⁴	9,37·10 ¹⁰

Disregarding the effects of gravity, we can write Eq. (51) as

$$\frac{d}{dx} \left(k \frac{dP}{dx} \right) = 0, \quad (53)$$

and its solution is

$$\Delta P_L = P - P_0 = c\beta. \quad (54)$$

An essential role in ensuring optimal operating conditions is played by the condenser section of a tubular heat exchanger. Its surface area must be sufficiently large to ensure that all the vapor fed to it condenses and the porous wick must have characteristics which will ensure that all the condensate is removed and carried to the evaporator.

Let us consider a simplified model of the wick in the condenser section of a tubular heat exchanger (Fig. 2). We assume the wick to be rigid and isotropic with a porosity factor Π , we also assume a constant temperature along the entire condenser length and, therefore, a constant pressure $P_V(c)$. The curvature radius of the vapor-liquid interface is R_C .

The condensate flows through the condenser wick at a mean velocity \bar{U}_L , while there is no hydrodynamic interaction between the vapor stream in the tube core and the condensate stream in the porous wick. The thermal flux is constant along the entire condenser.

We unroll the porous wick in the condenser into a flat plate and we consider the mass balance in an element of thickness dx , width b , and height a . The transverse cross section area is $S = ab$.

The stream of liquid leaving such an element is

$$I = \rho_L S \left(\bar{U}_L + \frac{d\bar{U}_L}{dx} dx \right). \quad (55)$$

As a result of condensation having occurred on surface S ,

$$I = I_x + I_V, \quad (56)$$

where $I_V = \rho_V U_V b dx$.

The equation of moments on this element in the x -direction is

$$\Delta M = \frac{\rho_L}{g} \cdot \frac{d(\bar{U}_L)^2}{dx} dx. \quad (57)$$

The moment on the liquid in a porous wick changes on account of friction and inertia forces acting on the flowing liquid, $\Sigma F = \Delta M$.

If the curvature radius of the vapor-liquid interface is R at the entrance to the element and $R + dR$ at the exit from the element, then the force driving the liquid - a result of the pressure gradient (an effect of the capillary potential) is

$$F = - \frac{2\sigma}{R} \cdot \frac{dR}{R} \Pi S. \quad (58)$$

This driving force is balanced by the friction forces

$$F_f = \Pi S \frac{dP}{dx} dx. \quad (59)$$

Assuming that the liquid flows through the wick according to Darcy (40), we have

$$kvI_L dx + S \frac{2\sigma}{R} \cdot \frac{dR}{R} = \frac{\rho_L}{g} \cdot \frac{d(U_L)^2}{dx} dx. \quad (60)$$

We now consider the heat balance in a condenser.

Let $Q_1 = I_L h_L$ be the quantity of heat supplied to surface S of a wick element by convection, $Q_2 = I_V h_V$ be the quantity of heat supplied to the wick surface bdx by vapor condensation, $Q_3 = qbdx$ be the quantity of heat carried off the outer surface bdx of the wick, and $Q_4 = I_L h_L + dI_L h_L/dx$ be the quantity of heat carried off the wick element through surface S . Then

$$Q_1 + Q_2 = Q_3 + Q_4, \quad (61)$$

$$I_L h_L + I_V h_V = qbdx + I_L h_L + \frac{d(I_L h_L)}{dx} dx. \quad (62)$$

After simple transformations, we have

$$\frac{dU_L}{dx} = \frac{qb}{\rho_L (h_V - h_L) \Pi S} = \frac{qb}{\rho_L \Pi S r}. \quad (63)$$

Integrating (63) yields

$$U_L = \frac{qb}{\rho_L \Pi S r} x, \quad (64)$$

and, since $I_L = \rho_L \Pi S U_L$,

$$I_L = \frac{qb}{r} x. \quad (65)$$

Inserting Eq. (64) into (60) and integrating, we obtain

$$\int_0^{l_c} kv \frac{qb}{r} x dx + \int_{R_c}^{R_i} S \frac{2\sigma}{R} \frac{dR}{R} = -2 \int_0^{l_c} \frac{1}{\rho_L g} \left(\frac{qb}{\Pi S r} \right)^2 x dx. \quad (66)$$

Considering that $R_c \rightarrow \infty$, we have after integration:

$$\frac{kqbv}{r} \cdot \frac{l_c^2}{2} - S \frac{2\sigma}{R_i} = -2 \frac{1}{\rho_L g} \left(\frac{qb}{\Pi S r} \right)^2 \frac{l_c^3}{2}, \quad (67)$$

$$l_c = \left[S \frac{2\sigma}{R_i} \frac{1}{\frac{kqbv}{r} + \frac{1}{\rho_L g} \left(\frac{qb}{\Pi S r} \right)^2} \right]^{-1/2}, \quad (68)$$

$$q = -k\rho_L g \frac{\Pi^2 S^2 r v}{2b} + \frac{\Pi S r}{b} \left(k^2 \rho_L^2 g^2 \Pi^2 S^2 v^2 + 2\sigma \rho_L g \frac{S}{R_i l_c^2} \right)^{1/2}, \quad (69)$$

$$Q_c = - \int_0^{l_c} qbdx = -k\rho_L g \Pi^2 S^2 r \frac{l_c^2}{2} - \Pi S r \left[(v l_c k \rho_L g \Pi S)^2 + 2\sigma \rho_L g \frac{S}{R_i} \right]^{1/2}. \quad (70)$$

If the energy and the mass balance in the evaporator section of a tubular heat exchanger are treated analogously, then one can determine the maximum evaporator length l_e for a given thermal flux density at the evaporator wall q or the maximum thermal power Q_{\max} which can be drawn from the evaporator with a given wick geometry. It is assumed here that evaporation takes place at the wick surface and that the heat is supplied to the evaporation zone by convection through the porous wall. Then

$$l_e = \left[\frac{4\sigma r S}{R_i q b \left(2 \frac{b}{r \rho_L S \Pi^2} + k \frac{\mu_L}{\rho_L} \right)} \right], \quad (71)$$

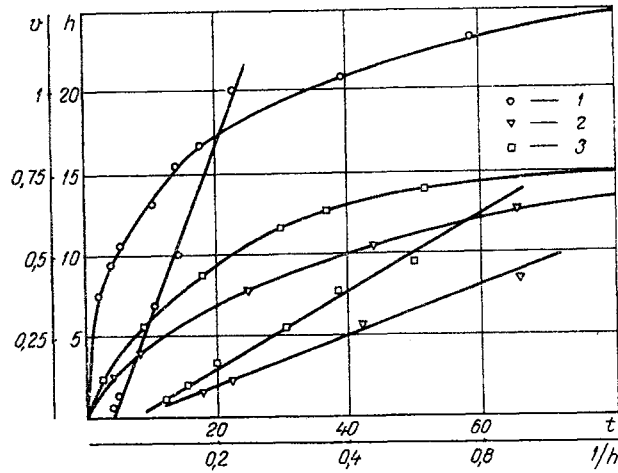


Fig. 4. Curves of capillary suction kinetics for various porous materials: 1) glass fiber wick; 2) Alundum wick with ethanol; 3) ethacryl wick with ethanol; v (cm/sec), h (cm), t (min), and $1/h$ (cm^{-1}).

$$Q_e = -k \frac{\mu_L}{4} q \Pi^2 S r l_e - l_e g \Pi^2 S \frac{r}{2} \left(\frac{k^2}{4} \mu_L l_e^2 - \frac{8 \rho_L \sigma}{g \Pi^2 R_i} \right)^{1/2}, \quad (72)$$

$$R_i = \frac{2 g_0 \sigma}{\rho_L g h_{\max}}$$

Usually R_i is found experimentally from the height of the liquid rise in the porous wick.

For a given thermal flux q and given wick dimensions (S , Π , b), therefore, we know the necessary condenser length l_c and evaporator length l_e . For the final determination of the thermal flux through a tube of given dimensions, however, one must take into account the length of the thermally insulated heat exchanger section and the tube position in the field of gravity.

For laminar vapor and liquid flow:

$$\Delta P_{\text{cap}} \geq \Delta P_V + \Delta P_L + \Delta P_B$$

$$= \left(1.234 + \frac{5.3}{\text{Re}_x} \cdot \frac{l_e}{R} \right) \frac{1}{\rho_V} j^2 + 8 \frac{l_i}{R \text{Re}_x} \frac{1}{\rho_V} j^2 + k l_V j + (\rho_L - \rho_V) g l \sin \alpha. \quad (73)$$

From this:

$$j = - \frac{k l \mu_L}{2M} \left(\frac{\rho_V}{\rho_L} \right) - \left[\frac{(k l \mu_L)^2 \left(\frac{\rho_V}{\rho_L} \right)^2}{8M^2} - \left[\frac{\left(1 - \frac{\rho_V}{\rho_L} \right) g l \sin \alpha + h_{\max} \frac{g}{g_0}}{M} \left(\frac{\rho_V}{\rho_L} \right) \right] \right]^{1/2}, \quad (74)$$

$$M = 1.234 + \frac{5.3 l_e + 8 l_i}{R \text{Re}_x},$$

$$l = l_e + l_c + l_i.$$

Accordingly, the thermal flux transmitted through the tube is

$$q = jr. \quad (75)$$

In order to study the heat and the mass transfer processes in low-temperature tubular heat exchangers, the authors have developed an experimental apparatus (Fig. 3) consisting of a quartz tube $l = 48$ cm, $d_1 = 37$ mm, $d_2 = 32$ mm, and fused at one end. An electric heater in an adiabatic case was placed at the fused end of the tube. A cooler – a heat exchanger – was placed at the other end of the tube. Liquid at a temperature of 10°C was fed into the heat exchanger from a precision thermostat. The power passing through the tube was measured with a wattmeter, power was also measured in the condenser for control

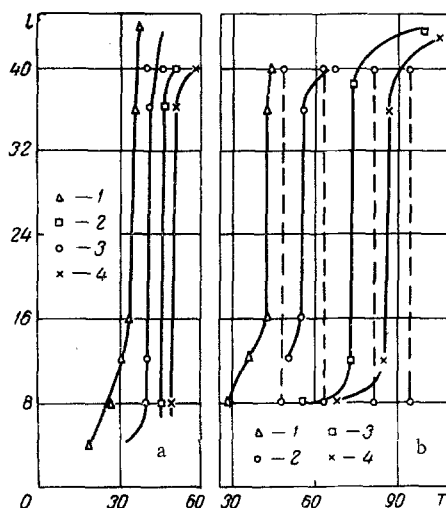


Fig. 5

Fig. 5. Temperature distribution along the tubular heat exchanger surface with: a) ethanol; 1) 5 W; 2) 13 W; 3) 18 W; 4) 20 W, power transmitted; b) water; 1) 6.5 W; 2) 28 W; 3) 75 W; 4) 125 W, power transmitted. (Solid lines refer to the tube surface, dashed lines refer to the vapor phase.) l (cm), T ($^{\circ}\text{C}$).

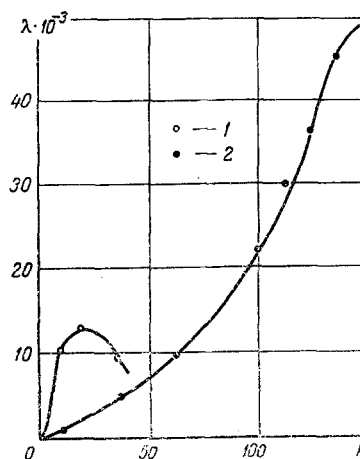


Fig. 6

Fig. 6. Effective thermal conductivity of tubular heat exchanger as a function of transmitted power: 1) ethanol; 2) water. λ ($\text{W}/\text{m} \cdot ^{\circ}\text{K}$), P (W).

purposes with thermocouples installed at the inlet and at the outlet. The power here was calculated by the formula:

$$Q = Gc\Delta t, \quad (76)$$

with Δt denoting the temperature drop.

The temperature distribution along the tube wall was measured with 11 copper-constantan thermocouples feeding signals into the automatic recorder.

The vapor pressure in the condenser was measured with a standard vacuumeter.

In order to reduce the heat losses, the tube was placed under a glass cover with the pressure reduced to 10^{-4} mm Hg. A metallic shield was placed inside the cover so as to suppress thermal radiation. Water and ethyl alcohol were used as the working liquids, their characteristics being listed in Table 1.

The wick was made of sintered glass fiber in the shape of a cylinder. It was fabricated as follows. Asbestos yarn was wound on a metal tube, then glass fiber on top of it along the tube making the outer diameter of the coil equal to the inner diameter of the quartz tube and the coil thickness 3 mm. The wick was then placed in a mold, to equalize the temperature distribution, and put in a shaft furnace at 800°C . After several minutes of heating the furnace was switched off and the wick was cooled in it for 4 h. Subsequently, the wick was ground and its ends cut off, it was then slipped off the metal tube for inspection of its capillarity and porosity as well as for measuring the thermal conductivity of the material. The final porosity was $\Pi = 29.9\%$, the maximum capillary rise was 20 cm for alcohol, and the fiber diameter was $10 \mu\text{m}$. Curves of suction kinetics for capillary-porous materials are shown in Fig. 4. An analysis of these curves will show that a wick with a longitudinal capillary structure (glass fiber) has better characteristics than a wick with a composite longitudinal-transverse capillary structure.

The tube for this experiment was prepared by a special technology. The liquid was poured into the tube held in a vertical position. Vapors of the liquid and uncondensable vapors in air were removed by a vacuum pump, whereupon the heater was switched on. The vacuum pump was run for 20 min. In order to prevent vapors of the liquid from entering the pump, a nitrogen trap was placed in front of the pump across the duct. After evacuation, the tube was hermetically sealed.

The experimental procedure was as follows. Thermal power was supplied to the evaporator intermittently. Each power shot was applied when a steady-state had been reached following the application of the preceding shot, and these steady-state steps were recorded on the instrument.

To the tube containing alcohol the power was applied in 2-W shots, to the tube containing water it was applied in 10-W shots. The temperature distribution along the tube as well as the vapor pressure in the tube were measured throughout the experiment. The vapor temperature in the tube was determined from the saturated vapor pressure. The temperature distributions on the outer surface of the quartz tube as well as the vapor pressures are shown in Fig. 5a, b.

The effective thermal conductivity of the heater tube at each power application was calculated according to the formula:

$$\lambda = QL/S\Delta t,$$

with Q denoting the thermal power in the condenser, L the length of tube between heater and condenser, S the tube cross section area, and Δt the temperature drop.

The effective thermal conductivity is shown in Fig. 6 as a function of power, for ethanol and water. It is evident here that the effective thermal conductivity of the tube is three times higher with water than with ethanol.

For comparison, the characteristics of various liquids used as heat carriers in low-temperature tubular heat exchangers are given in Table 1. On the basis of these data, one may conclude that water and ammonia are the most effective heat carriers.

In the range of helium temperatures the most effective heat carrier seems to be helium II below 2°K.

It ought to be noted, in conclusion, that the theory of tubular heat exchangers has to this day not been finalized yet. In order to verify the hypotheses proposed so far, one must have a sufficient amount of data available on a wide range of tube materials and heat carrier liquids.

NOTATION

l	is the length of heater tube;
d	is the outer diameter of tube;
P	is the pressure;
σ	is the liquid surface tension coefficient;
R_{\min}	is the minimum curvature radius;
U	is the velocity;
ρ	is the density;
μ, ν	are the dynamic and kinematic viscosity;
R	is the radius – the distance from the tube axis to the inner wick surface;
j	is the mass flow density in the tube;
q	is the heat flow density in the tube;
K	is the permeability of porous wick in the tube;
θ	is the mass content in the wick;
Φ	is the capillary potential;
τ	is the propagation velocity of the mass front in the porous wick;
h	is the altitude above sea level;
I	is the mass flow in the tube;
Q	is the thermal flux in the tube;
Π	is the porosity of the wick;
g_0	is the acceleration of free fall;
g	is the gravitational constant;
G	is the rate of liquid flow;
c	is the specific heat;
Δt	is the temperature drop.

Subscripts

V	denotes vapor;
L	denotes liquid;

e denotes evaporator;
c denotes condenser;
i denotes thermally insulated zone;
cap denotes capillary;
f denotes friction;
r denotes radial coordinate;
x denotes axial coordinate;
a. r denotes aerodynamic resistance;
in denotes inertia.

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